Abstract Education

MATHEMATICS

NUMBER

Abstract Education First Floor, Town Hall, Brixham, Devon, TQ5 8TA

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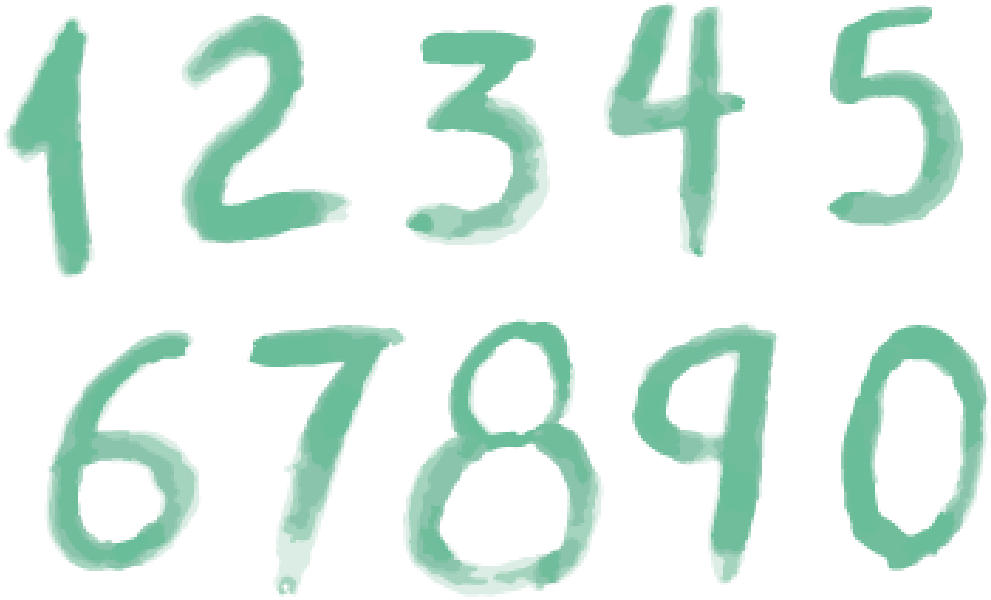
WHY?

Numbers and number systems.

A number is a basic unit of mathematics. You will use numbers for everyday tasks such as for counting, measuring, and comparing amounts. A number system is a set of symbols, or numerals, that are used to represent numbers.

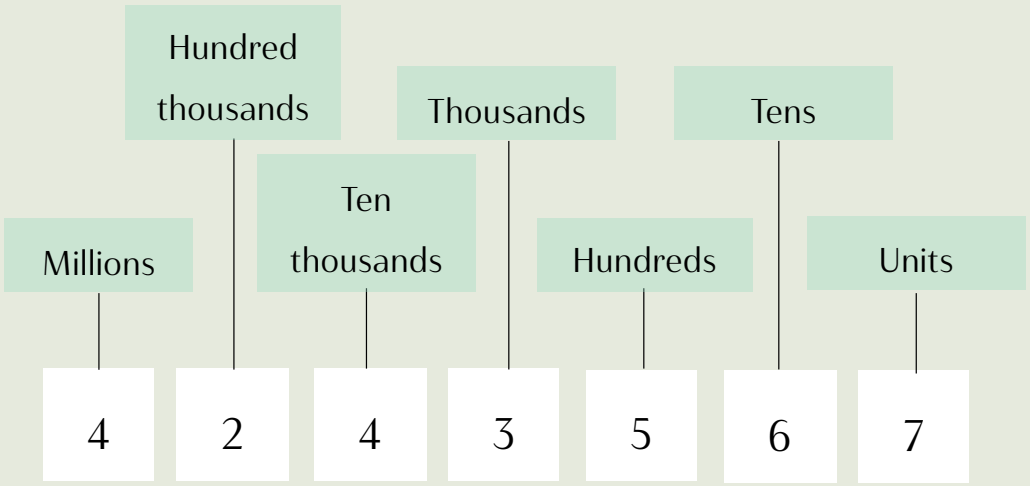
The most common number system uses 10 symbols called digits–0, 1, 2, 3, 4, 5, 6, 7, 8 and 9–and combinations of these digits.



PLACE VALUE

Place value refers to the value of each digit within a number.

Let's look at the number 4243567



In the number 4243567 the digit 5 has the value of 500 (five hundred), and the digit 4 has a value of 4,000,000 (four million).

Another example, the 5 in 350 represents 5 tens, or 50; however, the 5 in 5,006 represents 5 thousands, or 5,000.

It is important to understand that whilst a digit can be the same, its value depends on where it is in the number.

PLACE VALUE

Place value

A number can be made of more than one digit. So we use place value headings, like ones, tens, hundreds and thousands, to help us understand the value of each digit.

Ten thousands	10 000
Thousands	1 000
Hundreds	100
Tens	10
Units	1

Each time we want to show a bigger number, we just add one column to the left and we know it is always 10 times bigger than than the column on its right.

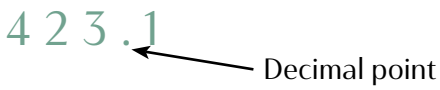
Tens are 10 times bigger than ones

Hundreds are 10 times bigger than tens

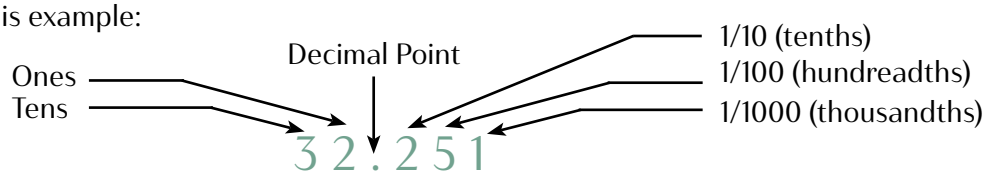
As we move right, each position is 10 times smaller. From hundreds, to tens, to ones.

But what if we continue past ones? What is 10 times smaller than ones?

1/10 ths are 10 times smaller than ones but we must first put a decimal point, so we know exactly where the ones position is:



We can continue with smaller and smaller values, from tenths, to hundredths, and so on, like in this example:



PLACE VALUE

Ordering and comparing numbers

Which is the largest number, 3562 or 3496?

To put the numbers in order of size, you compare the digits by place value.

To work this out, line the numbers up in a table by place value.

Start from the left-hand side with the digit which has the largest place value. The digit in the thousands column is the same, so you look at the next digit, the hundreds. Here the 5 is greater than the 4, so 3562 is the larger number.

Notice that you do not need to compare the rest of the digits once you have found one place value where the digit is larger.

Thousands (Th)	Hundreds (H)	Tens (T)	Units (U)
3	5	6	2
3	4	9	6

Which is the larger number, 32.76 or 32.8?

Tens (T)	Units (U)	.	Tenths	Hundredths
3	2	.	7	6
3	2	.	8	

Here again the digit with the largest place value, the tens, is the same for both numbers, as is the units digit. Moving to the tenths column, 8 is greater than 7, so 32.8 is the larger number.

PLACE VALUE

Let's look at one more example. Which is the larger number, 8,825.5 or 21,812?

Ten Thousands	Thousands	Hundreds	Tens	Units	.	Tenths
	8	8	2	5	.	5
2	1	8	1	2	.	

In this example it is clear that 21,812 is the larger number, as the first digit for 8,825.5 is in the thousands column, whereas the first digit for 21,812 is in the ten thousands column.

NEGATIVE NUMBERS

Negative numbers have many uses in everyday life. For example:

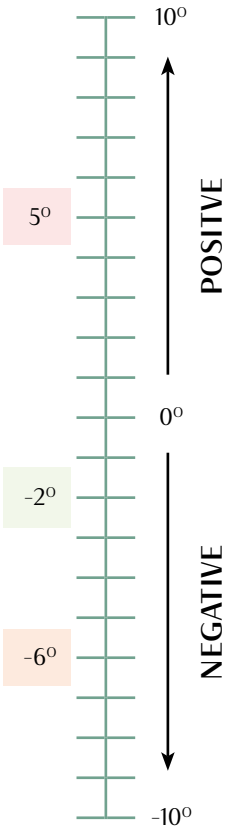
- Money withdrawn from a bank account
- Indicate temperatures below zero
- When measuring sea levels below a certain point
- An elavotor or lift may take you to floors below zero such the lower ground floor or parking.
- Some sports use negative numbers when calculating scores, for example, golf.

There are a few rules which are helpful when working with negative numbers:

- A negative number is a number which is less than zero.
- A negative number has a minus sign at the beginning. If there is no minus sign, it is not a negative number.
- A decimal number such as 0.75 is not a negative number, it is just a small number which is obviously less than 1.

NEGATIVE NUMBERS

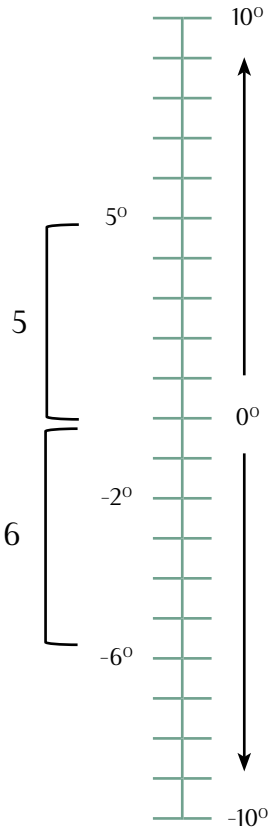
When working with negative numbers and how to represent them, it helps to do so on a number line. Anything above the zero in this number line is a positive number and anything below the zero is a negative number. Let's mark 5°C , -2°C, -6°C on the number line.



If we begin with 5 degree Celsius, which is not a negative number, this will be placed above the zero. Therefore we count 5 spaces above the 0.

Minus 2 degrees celsius is a negative number because we have a minus sign at the beginning. We count downward this time from zero.

For minus 6 degrees celsius we count 6 spaces down from 0.



You can see the lowest temperature is –6 °C and the highest temperature is 5 °C, but what is the difference between these two temperatures?

If you count up from –6 on the scale, you need to go up by 6°C to get to zero and then count up another 5 to get to 5 °C.

This means the difference between the temperatures is:

$$6 + 5 = 11\ ^{o}C$$

So, because one of the temperatures is positive and one is negative you need to add 6 and 5.

NEGATIVE NUMBERS

What is the difference between –6 °C and –2 °C?

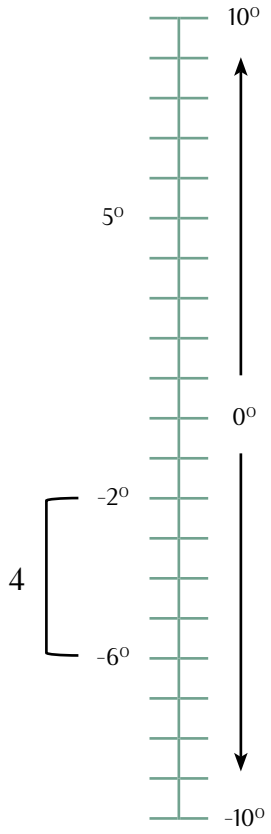
If you count up from –6 on the scale, you need to count up by 4°C to get to –2 °C.

This means the difference between the temperatures are:

$$6 - 2 = 4°C$$

So, because both of the temperatures are negative you need to find the difference between 6 and 2.

This means you can ignore the minus signs and take the smaller number from the larger number.



NEGATIVE NUMBERS

Example

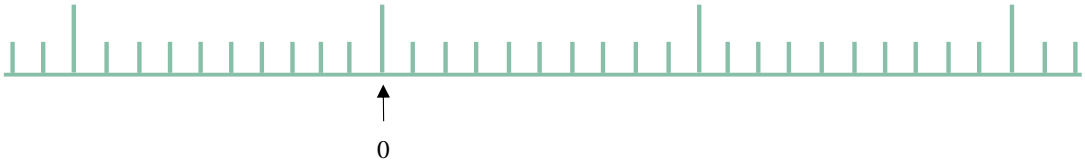
Now we will look at an example.

The table shows the places Charlie visited and their height above sea level.

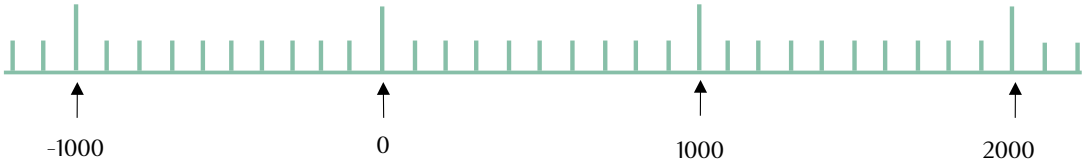
Place	Height above sea level
Caspian Sea	-28 m
Ben Nevis	1345 m
Great Gable	899 m
Dead Sea	-431 m
Snowdonia	1085 m

Put the heights of places above sea level in order of size, from the lowest to the highest.

First we will mark zero on the number line. Remember we have two numbers below zero, this means below sea level.

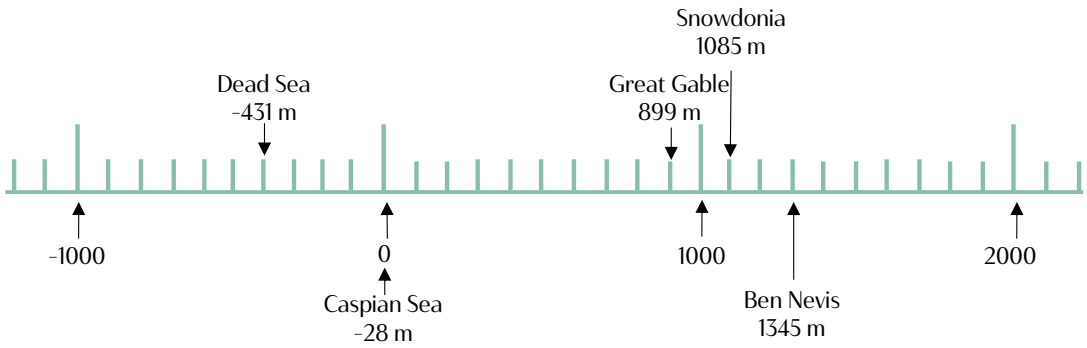


As we are counting in thousands, mark these on the number line so all of your numbers fit on. The number line will not show each of your numbers accurately, but you can put them on the number line close to where they should lie.



NEGATIVE NUMBERS

Begin by putting the positive numbers on the number line followed by the negative heights below zero.



The heights in order of size are:
–431 m
–28 m
899 m
1085 m
1345 m

What is the difference between the heights of the highest and lowest places?

Highest: Ben Nevis 1345 m
Lowest: Dead Sea –431 m

As one is below sea level and the other is above sea level, the difference in height is:

$$1345 + 431 = 1776 \text{ m}$$

NUMBERS

Common types of numbers:

Natural numbers

Natural numbers - or the counting numbers - are probably the first type of numbers you will have come across. The Natural numbers are whole numbers greater than zero. These numbers are:

1, 2, 3, 4, ..., numbers that you can physically count

The Natural Numbers do not include negative values. For these we need another group.

Integers

Integers are whole numbers, both negative and positive, including zero. These numbers are:

Positive integers ← Negative integers →

-3, -2, -1, 0, 1, 2, 3, ...

Positive integers are integers greater than zero, and negative integers, not surprisingly, are integers less than zero.

Rational numbers

Rational number are numbers that can be written as a fraction where both the numerator and denominator are integers. This means that both the top and bottom of the fraction are whole numbers. Examples of these would be ⅓ and 23/7.

Irrational Numbers

Irrational Numbers are numbers that can't be written as fractions, examples of these are ones such as π and √2. If we try to write these numbers as decimals they go on forever, with no recurring digits.

Square Numbers

Square Numbers are integers that can be written as the square of some other

integers, ie. a product of an integer multiplied by itself. Examples of these are 4, (2×2) and 81, (9×9). Square Numbers can also be written in the form 5^2. This notation means 5 squared, which is 25.

Surds

Surds are numbers left in the form $\sqrt{n}$, where n is a positive integer that is not a square number.

Prime Numbers

Prime numbers are numbers greater than 1 that can only be divided by themselves and 1 to give an integer answer. Examples of these are:

2, 3, 5, 7, 11, 13, 17, ...

Note that 2 is the first and only even prime number.

Real Numbers

Real numbers are all the numbers that you have ever come across, all of the rational and irrational numbers. All of these real numbers can be written in a finite or infinite decimal form, such as:

$\sqrt{2}$ = 1.414213562 ... and π = 3.141592654

Types of Numbers

Cube numbers

A cube number is a number multiplied by itself 3 times.

The number 1 can be written as 1 x 1 x 1 or 1^3

The number 8 can be written as 2 x 2 x 2 or 2^2

27 can be written as 3 x 3 x 3 or 3^3

NUMBERS

When an integer is multiplied by itself and then by itself again, the result is a cube number. In the examples above 1, 8 and 27 are all cube numbers.

Factors

The factors of 12 are all the numbers which will divide exactly into 12:

1, 2, 3, 4, 6 and 12.

Prime factors

The factors of 12 are 1, 2, 3, 4, 6 and 12. Of these, 2 and 3 are prime numbers, so 2 and 3 are the prime factors of 12.

Highest common factor

The factors of 24 are 1, 2, 3, 4, 6, 8, 12 and 24 and that the factors of 40 are 1, 2, 4, 5, 8, 10, 20 and 40.

The numbers 1, 2, 4 and 8 appear in both lists, so they are known as the common factors of 24 and 40.

The number 8 is the highest of them, and it is called the highest common factor (HCF). The 1 is the lowest of them, and is called the lowest common factor (LCF).

Multiples

The multiples of a number are all the numbers that it will divide into. This includes the number itself. For example:

The multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16...

The multiples of 5 are 5, 10, 15, 20, 25, 30, 35...

The multiples of 20 are 20, 40, 60, 80, 100...

Multiples are just like times tables.

NUMBERS

Square roots

The opposite of a square number is a square root. We use the symbol √ to mean square root.

So we can say the square root of √4 = 2 and the square root of √25 = 5.

However, this is not the whole story, because -2 x -2 is also 4, and -5 x -5 is also 25. So, in fact, √4 = 2 or -2. And √25 = 5 or -5.

Remember that every positive number has two square roots.

Cube roots

The opposite of a cube number is a cube root. We use the $\sqrt[3]{}$ symbol to mean cube root.

So $\sqrt[3]{8}$ is 2 and $\sqrt[3]{27}$ is 3.

Each number only has one cube root.

Directed numbers

Numbers can either be positive or negative. Often brackets are put around negative numbers to make them easier to read, e.g. (-2). If a number is positive, the + is usually missed out before the number. So 3 is really (+3). Adding and multiplying combinations of positive and negative numbers can cause confusion and so care must be taken.

NUMBERS

Accuracy

Approximation

An approximation is anything that is similar but not exactly equal to something else. The common types of approximation are dealt with below.

Rounding

It is not always necessary to give the exact number. For example, if there were 54,776 inhabitants in a town, you could say that its population was approximately 55,000.

By doing this we have rounded 54,776 to the nearest thousand to give 55,000.

Upper and lower bounds

Numbers can be written to different degrees of accuracy. For example:

4.5, 4.50 or 4.500

Although appearing to represent the same number, they don't. This is because they are written to different degrees of accuracy.

If we are told that a piece of wood is 12cm long to the nearest cm, then what is the range of possible lengths it could be?

It must be at least 11.5cm long to round up to 12cm. But it must be less than 12.5cm, to round down to 12cm. If it was 12.5cm or more we would round up to 13cm.

Since 12.5cm is the upper limit we call this the 'upper bound'. So we say that 11.5cm is the lower bound and 12.5cm is the upper bound for the length of the piece of wood.

Example 1: A fence is 22m long to the nearest metre. What are the upper and lower bounds?

The lower bound is 21.5m and the upper bound is 22.5m

Example 2: A box is 8.5cm wide measured to the nearest tenth of a cm. What are the upper and lower bounds?

Anything above 8.45cm will round up to 8.5 to the nearest tenth of a cm. Anything less than 8.55cm will round down to 8.5cm The lower bound is 8.45cm and the upper bound is 8.55cm

Calculating with upper and lower bounds

When numbers are written to a specific degree of accuracy, calculations involving those numbers also give a range of possible answers.

In these calculations we need to think about:

- whether we need upper or lower bounds
- whether to add, subtract, multiply or divide
- how sensible our answers are

The order of operations

BIDMAS stands for:

brackets
indices
division
multiplication
addition
subtraction

The order in which we carry out a calculation is important.

NUMBERS

Example: What is 2 + 3 x 4?

If we calculate the '2 + 3' part first, we get:

$$2 + 3 = 5$$

If we then multiply the answer by 4:

$$5 \times 4 = 20$$

Our answer is 20.

This is NOT correct!

BIDMAS tells us that 'multiplication' comes before 'addition', If we calculate the '3 x 4' part first, we get:

$$3 \times 4 = 12$$

Then add 2 to your answer:

$$2 + 12 = 14$$

so the second answer is correct which is 14.

NUMBERS

Standard index form

Calculations with very big or small numbers can be made easier by converting numbers in and out of standard form.

In standard form, a number is always written as: $A \times 10^{n}$

A is always between 1 and 10. n tells us how many places to move the decimal point.

To convert a number into standard form, split the number into two parts - a number multiplied by a power of 10.

Powers of 10

Standard form uses the fact that the decimal place value system is based on powers of 10:

$10^{0} = 1$
$10^{1} = 10$
$10^{2} = 100$
$10^{3} = 1000$
$10^{4} = 10000$
$10^{5} = 100000$
$10^{6} = 1000000$

Example 1: Write 50,000 in standard form. 50,000 can be written as:

5 x 10,000

10,000 = 10 x 10 x 10 x 10 = 10^{4}

So: 50,000 = 5 x 10^{4}

NUMBERS

Example 2: Write 15 000 000 in standard index form.

$15\ 000\ 000 = 1.5 \times 10\ 000\ 000$

This can be rewritten as:

$$1.5 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10$$

$$= 1.5 \times 10^{7}$$

You can convert from standard form to ordinary numbers, and back again. Have a look at these examples:

$3 \times 10^{4} = 3 \times 10\ 000 = 30\ 000$ (Since $10^{4} = 10 \times 10 \times 10 \times 10 = 10\ 000$)

$2\ 850\ 000 = 2.85 \times 1\ 000\ 000 = 2.85 \times 10^{6}$ Make the first number between 1 and 10.

$$0.000467 = 4.67 \times 0.0001 = 4.67 \times 10^{-4}$$

Adding and subtracting numbers in standard index form

Convert them into ordinary numbers, do the calculation, then change them back if you want the answer in standard form.

Example: $4.5 \times 10^{4} + 6.45 \times 10^{5}$

$$= 45{,}000 + 645{,}000$$

$$= 690{,}000$$

$$= 6.9 \times 10^{5}$$

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Multiplying and dividing numbers in standard form

Here you can use the rules for multiplying and dividing powers. Remember these rules:

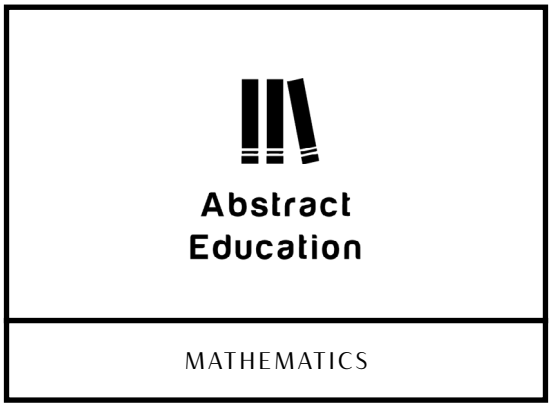
To multiply powers you add: $10^5 \times 10^3 = 10^8$

To divide powers you subtract: $10^5 \div 10^3 = 10^2$

Example: Simplify $(2 \times 10^3) \times (3 \times 10^6)$

Multiply 2 by 3 and add the powers of 10:

$$(2 \times 10^3) \times (3 \times 10^6) = 6 \times 10^9$$



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